Trajectory and movement planning of a perpendicular parking maneuver for an autonomous driving model car

Nico Schick[1]

The perpendicular parking is an essential discipline for an autonomous car. This includes also a suitable trajectory and movement planning as prerequisite. In this publication, such a mathematical-based design is described for an autonomous model car.

I. Motivation

The technology in the automotive industry is successively evolving. Autonomous driving plays an important role here. Along with the mechanical development of the vehicle, there is also a technical development - with the latest sensor technologies and software algorithms. In the future, the vehicle itself could take over the control. The perpendicular parking is one of the core disciplines for an autonomous car. In general, this parking movement can be divided into three consecutive sub tasks. At first, a free perpendicular parking lot must be detected by an image processing method (e. g. blob analysis or neural network). Afterwards, a trajectory and movement planning has to be defined. Finally, the car can follow the trajectory with the use of a suitable lateral controller (e. g. PID governor, state space controller, Stanley approach). In this publication, a regarding trajectory and movement planning for an autonomous model car will be illustrated. [1] [2] [3] [4]

II. Trajectory planning

There are several possibilities to develop a suitable trajectory for the perpendicular parking maneuver. One of them is given with the combination of a straight line and a quadrant. The regarding build-up is visualized in Figure 1. Here, the perpendicular parking lot is considered to be on the left-hand side of the direction of travel. The complete consideration is in world coordinates which are defined by the global coordinates x_w and y_w. The starting point of the vehicle respectively of the straight line is at point $S_0(\, x_{w,0} \,|\, y_{w,0} \,)$. The straight line has a length of s_l. The vehicle shall drive on this straight with its initial and constant velocity v_0. Afterwards, the quadrant respectively the cornering starts at point $S_c(\, x_{w,c} \,|\, y_{w,c} \,)$. The vehicle starts to decelerate at point $S_B(\, x_{w,B} \,|\, y_{w,B} \,)$. Finally, the car shall stop at its center of gravity respectively at the final position defined by $S_E(\, x_{w,E} \,|\, y_{w,E} \,)$. Furthermore, the circle mid point is given by $S_K(\, x_{w,K} \,|\, y_{w,K} \,)$ and the corresponding radius with R_K. The mid point of the perpendicular parking lot is named by $S_M(\, x_{w,M} \,|\, y_{w,M} \,)$. With respect to the car itself, the wheelbase is defined by l as well as the distance between S_E and the vehicle's rear axle with l_h. The complete trajectory is named by S_T. In the following, the corresponding mathematical descriptions are explained in more detail. [5]

In the underlying case, the vehicle's center of gravity is not at the same coordinates as the mid point of the perpendicular parking lot. But the vehicle shall clearly stop at the mid of the parking lot. As a consequence, the size l_e was introduced. This length is used to compensate the coordinate deviation of S_E and S_M accordingly

$$l_e = \frac{l}{2} - l_h \tag{1}$$

With the help of l_e, the end coordinate of the trajectory in vertical direction can be determined in the following way

$$y_{w,E} = y_{w,M} - l_e \tag{2}$$

A circle is mathematically clearly described by its mid point and the radius. They are calculated by geometric equations as follows

$$R_K = y_{w,E} - y_{w,0} \tag{3}$$

$$x_{w,K} = x_{w,E} - R_K \tag{4}$$

$$y_{w,K} = y_{w,E} \tag{5}$$

Moreover, the first coordinate of the quadrant in horizontal direction can be set to

$$x_{w,c} := x_{w,K} \tag{6}$$

The total trajectory depends on the regarding horizontal and vertical coordinates. To be precise, it can be written as follows

$$S_T = \{\ (x_{w,T}(\Gamma,\Omega),\ y_{w,T}(\Gamma,\Omega))\ \} \tag{7}$$

The radian range is defined by $\Omega \in \left[\frac{3}{2}\pi\,;\,2\pi\right]$. The horizontal and vertical coordinates themselves are defined with

$$x_{w,T}(\Gamma,\Omega) = \begin{cases} \Gamma, & \Gamma \in [x_{w,0}\,;\,x_{w,c}] \\ R_K\,\cos(\Omega) + x_{w,K}, & \Gamma \in (x_{w,c}\,;\,x_{w,E}] \end{cases} \tag{8}$$

$$y_{w,T}(\Gamma,\Omega) = \begin{cases} y_{w,0}, & \Gamma \in [x_{w,0}\,;\,x_{w,c}] \\ R_K\,\sin(\Omega) + y_{w,K}, & \Gamma \in (x_{w,c}\,;\,x_{w,E}] \end{cases} \tag{9}$$

Furthermore, the straight line of the trajectory can be determined as follows

$$s_l = x_{w,c} - x_{w,0} \tag{10}$$

[1]N. Schick, M. Sc. studied Applied Informatics (M. Sc.) and Technical Informatics (B. Eng.) at the University of Applied Sciences in Esslingen.

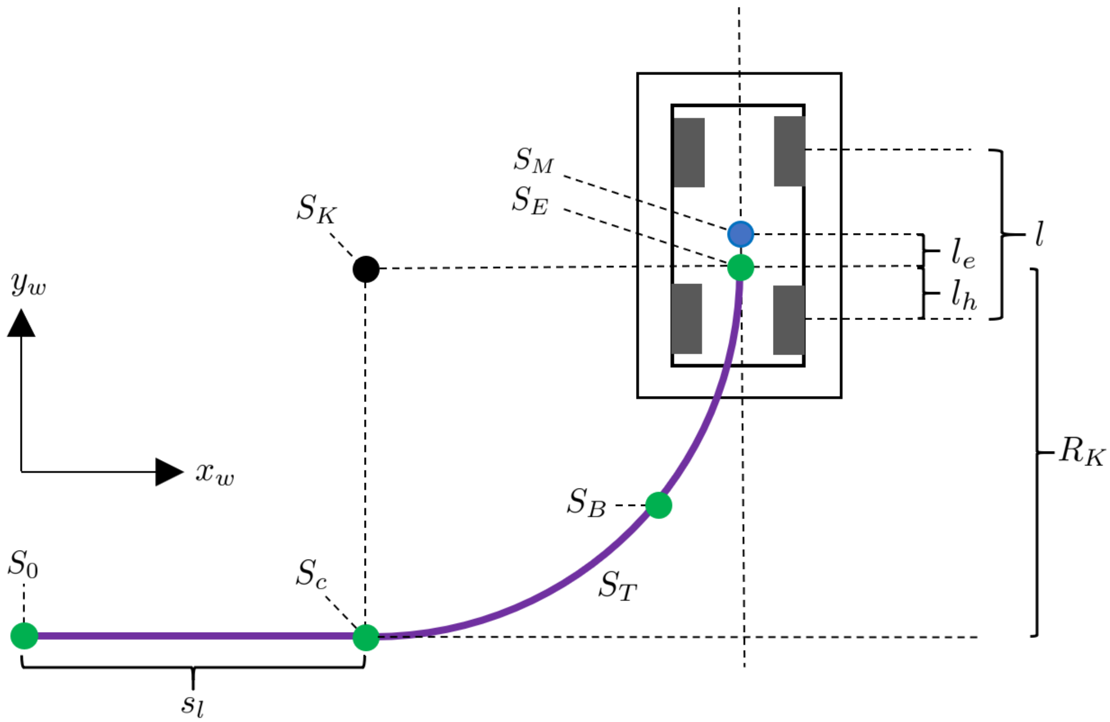

Fig. 1. Visualization of the trajectory and movement planning

The length of the quadrant is defined by the quarter of the complete arc length

$$b_S = \frac{\pi}{2} R_K \tag{11}$$

As a result, the complete distance of the trajectory can be calculated in the following way

$$s_{ges} = s_l + b_S \tag{12}$$

The distance on the quadrant with constant velocity is named by b_0 and with decelerated movement with b_a. Here, both sizes are the result of the splitting of b_S with the help of the parameter $\lambda \in (0\,;\,1)$

$$b_0 = \lambda\, b_S \tag{13}$$

$$b_B = b_S - b_0 \tag{14}$$

The higher the initial velocity v_0, the lower λ should be chosen to really stop at the intended final position within the parking lot.

With respect to the implementation, the number of discretization points for the trajectory and movement dependent sizes must be defined. The regarding total number is named by N_{ges}. This value depends on the complete distance of the trajectory and the distance $p \in \mathrm{N}$ between each discretization point respectively

$$N_{ges} = \left\lceil \left(\frac{s_{ges}}{p} \right) \right\rceil \tag{15}$$

With the help of N_{ges}, the number of discretization points for the straight line (N_l), the drive on the quadrant with constant speed (N_0) as well as for the decelerated movement (N_B) can be determined

$$N_l = \left\lceil \left(\frac{s_l}{s_{ges}} N_{ges} \right) \right\rceil \tag{16}$$

$$N_0 = \left\lceil \left(\frac{b_0}{s_{ges}} N_{ges} \right) \right\rceil \tag{17}$$

$$N_B = N_{ges} - N_l - N_0 \tag{18}$$

III. Speed Planning

Beside the trajectory planning, the determination of suitable velocities is important also. The vehicle shall drive with constant velocity v_0 up to braking period. Afterwards, the vehicle's velocity is reducing as a result of the deceleration. The velocity values during braking can be determined by fundamental kinematics

$$v_B = v_0 - a_B\, t = \sqrt{{v_0}^2 - 2\, a_B\, b_B}, \quad t \in [0 \ldots t_B] \tag{19}$$

with a_B as the deceleration

$$a_B = \frac{{v_0}^2}{2\, b_B} \tag{20}$$

and t_B as the braking time

$$t_B = \frac{v_0 - \sqrt{{v_0}^2 - 2\, a_B\, b_B}}{a_B} \tag{21}$$

In total, the overall vehicle's velocity is written as follows

$$v_T(\Gamma) = \begin{cases} v_0, & \Gamma \in [x_{w,0}\,;\,x_B] \\ v_B, & \Gamma \in (x_B\,;\,x_{W,E}] \end{cases} \tag{22}$$

IV. Lights planning

During the complete perpendicular parking maneuver, the left blinker and braking lights of the vehicle shall be actuated also. The regarding signal names are as follows:

- $z_{Bl,lf}$: Blinker light at left front
- $z_{Br,lr}$: Braking light at left rear
- $z_{Br,rr}$: Braking light at right rear

The actuation timing of these lights are as usual. With respect to this, the blinker light at left front shall be flashed during cornering. Furthermore, the braking lights must be activated during the braking period.

V. Perpendicular parking map

The complete trajectory and movement planning for the perpendicular parking results in a regarding map. It contains the desired values at each step (Idx) for the coordinates of the trajectory, the vehicle's velocity as well as the left blinker and braking lights (see Figure 2). The desired coordinates of the trajectory are needed by the lateral controller, the vehicle's velocity by the longitudinal controller to control the vehicle speed and the actuation signals of the left blinker and braking lights by regarding actuating components of the car. Furthermore, the map was fully transformed into a SW function and integrated into a model car afterwards. Finally, the suitability of the map was demonstrated by real tests.

Idx	$x_{w,T}$	$y_{w,T}$	v_T	$z_{Bl,lf}$	$z_{Br,lr}$	$z_{Br,rr}$
1	$x_{w,0}$	$y_{w,0}$	v_0	0	0	0
...	...	...	v_0	0	0	0
$N_l + 1$	$x_{w,c}$	$y_{w,c}$	v_0	1	0	0
...	...	...	v_0	1	0	0
$N_l + N_0 + 1$	$x_{w,B}$	$y_{w,B}$	v_0	1	1	1
...	...	...	...	1	1	1
N_{ges}	$x_{w,E}$	$y_{w,E}$	0	0	0	0

Fig. 2. Map containing the trajectory and movement planning values

VI. Conclusion

Autonomous driving is already an essential technology in the automotive sector. One of the most challenging scenarios is given by a perpendicular parking maneuver. With respect to this, a suitable trajectory and movement planning is required. There are several design pattern for such a planning determination. One of them is described here with respect to an autonomous model car. Finally, its applicability was proven by real perpendicular parking tests.

References

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